Class - X Session 2022-23
Subject - Mathematics (Basic)
Sample Question Paper - 21

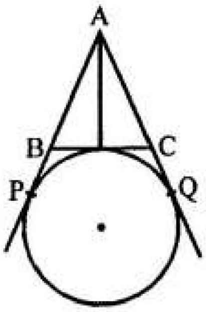
Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm then the length of AP is **[1]**



a) 7.5 cm b) 15 cm

c) 9 cm d) 10 cm

2. If the distance between the points (4, p) and (1,0) is 5, then the value of p is **[1]**

a) 0 b) 4 only

c) -4 only d) ±4

3. The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by **[1]**

a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ b) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$

c) $\left(\frac{x_1-y_1}{2}, \frac{x_2-y_2}{2}\right)$

d) $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$

4. If three coins are tossed simultaneously, then the probability of getting at least two heads, is [1]

a) $\frac{1}{2}$

b) $\frac{3}{8}$

c) $\frac{1}{4}$

d) $\frac{7}{4}$

5. If the point R(x, y) divides the join of P(x_1, y_1) and Q(x_2, y_2) internally in the given ratio $m_1 : m_2$, then the coordinates of the point R are [1]

a) $\left(\frac{m_2x_1-m_1x_2}{m_1+m_2}, \frac{m_2y_1-m_1y_2}{m_1+m_2}\right)$

b) $\left(\frac{m_2x_1-m_1x_2}{m_1-m_2}, \frac{m_2y_1-m_1y_2}{m_1-m_2}\right)$

c) $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

d) None of these

6. Graphically, the pair of equations 6x - 3y + 10 = 0, 2x - y + 9 = 0 represents two lines which are [1]

a) parallel

b) Intersect at two points

c) coincident

d) intersect at a point

7. From the letters of the word MOBILE, a letter is selected. The probability that the letter is a vowel, is [1]

a) $\frac{3}{7}$

b) $\frac{1}{6}$

c) $\frac{1}{2}$

d) $\frac{1}{3}$

8. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal then the ratio of its radius and the slant height of the conical part is [1]

a) 4 : 1

b) 1 : 4

c) 1 : 2

d) 2 : 1

9. In a lottery, there are 8 prizes and 16 blanks. What is the probability of getting a prize? [1]

a) $\frac{1}{2}$

b) None of these

c) $\frac{1}{3}$

d) $\frac{2}{3}$

10. If α and β are the roots of $ax^2 + bx + c = 0$, then the wrong statement is [1]

a) $\alpha^2 + \beta^2 = \frac{b^2-2ac}{a^2}$

b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{c}$

c) $\alpha+\beta=\frac{b}{a}$

d) $\alpha\beta=\frac{c}{a}$

11. $4x^2 - 2x - 3 = 0$ have **[1]**

a) Real roots

b) Real and Distinct roots

c) No Real roots

d) Real and Equal roots

12. If $\sqrt{3}\tan\theta = 3\sin\theta$, then the value of $\sin^2\theta - \cos^2\theta$ is **[1]**

a) 1

b) $\frac{1}{2}$

c) 0

d) $\frac{1}{3}$

13. Points (1, 0) and (-1, 0) lies on **[1]**

a) line x + y = 0

b) y-axis

c) x-axis

d) line x - y = 0

14. 2.35 is **[1]**

a) an integer

b) a rational number

c) an irrational number

d) a natural number

15. Construction of cumulative frequency table is useful to determine **[1]**

a) mean

b) all the three

c) median

d) mode

16. If the length of a shadow of a tower is increasing, then the angle of elevation of the sun is **[1]**

a) neither increasing nor decreasing

b) zero

c) decreasing

d) increasing

17. The HCF and the LCM of 12, 21, 15 respectively are: **[1]**

a) 3, 140

b) 420, 3

c) 12, 420

d) 3, 420

18. **Assertion (A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162 **[1]**
Reason: If a, b are two positive integers, then H.C.F. $\times$ L.C.M. = a $\times$ b

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. The value of 'k' for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has a non zero solution is **[1]**

a) 0
b) 8
c) 2
d) 6

20. **Assertion (A):** If two triangles are similar then they are congruent also. **[1]**
Reason (R): Ratio of perimeters of two triangles is always equal to ratio of their corresponding sides, medians, altitudes and angle bisectors.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. i. A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective? **[2]**
ii. Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

22. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically by the method of substitution. **[2]**

OR

Half the perimeter of a rectangle garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

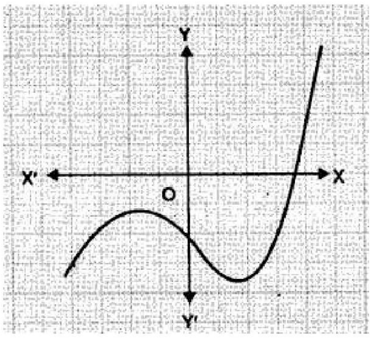
23. Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3. **[2]**

24. Two concentric circles of radii a and b (a >b) are given. Find the length of the chord of the larger circle which touches the smaller circle. **[2]**

OR

O is the centre of a circle of radius 8 cm. The tangent at a point A on the circle cuts a line through O at B such that AB = 15 cm. Find OB.

25. Find the number of zeroes of p(x). The graph of y = p(x) is given in figure below, for some polynomial p(x): **[2]**



Section C

26. Evaluate: $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$ **[3]**

27. Prove that $7\sqrt{5}$ is irrational. **[3]**

OR

In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

28. Form the pair of linear equations for the problem and find its solution by substitution method: **[3]**
The difference between the two numbers is 26 and one number is three times the other.

29. O is the centre of a circle of radius 5 cm. T is a point such that OT =13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E. Find length of AB. **[3]**

OR

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

30. Diagonal AC and BD of a trapezium ABCD with $AB||DC$ intersect each other at point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$. **[3]**

31. Two boats approach a light house in mid-sea from opposite directions. The angles of elevations of the top of the lighthouse from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the lighthouse. **[3]**

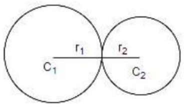
Section D

32. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train. **[5]**

OR

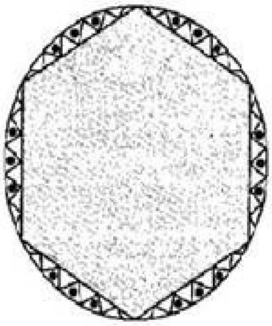
If the price of a book is reduced by ₹5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

33. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers? **[5]**



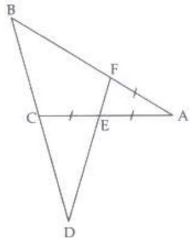
OR

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm ^{2}. (use $\sqrt{3}$ = 1.7)



34. In the given figure, line segment DF intersect the side AC of a triangle △ABC at the point E such that E is the mid–point of CA and ∠AEF = ∠AFE. Prove that: $\frac{BD}{CD} = \frac{BF}{CE}$. **[5]**

[Hint: Take point G on AB such that CG || DF.]



35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. **[5]**

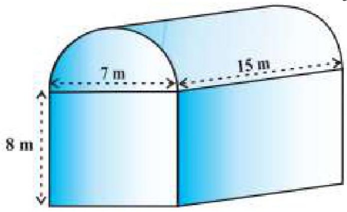
Class interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7

900-1000	4

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.
The diameter of the half cylinder was 7 m and length was 15 m.



(i) Find the volume of the air that the shed can hold.

(ii) If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space on an average, how much air would be in the shed when it is working?

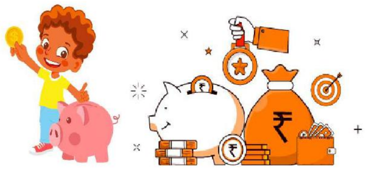
(iii) Find the surface area of the cuboidal part.

OR

Find the surface area of the cylindrical part.

37. **Read the text carefully and answer the questions:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

(iii) How much money Akshar saves in 10 days?

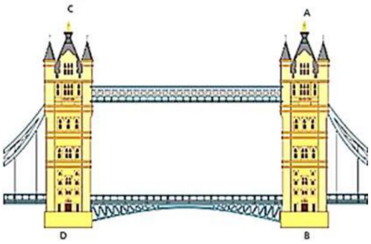
OR

How many coins are there in piggy bank on 15th day?

38. **Read the text carefully and answer the questions:** **[4]**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60^{o} and 30^{o} respectively.



(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

(iii) Find the distance between Neeta and top of tower AB?

OR

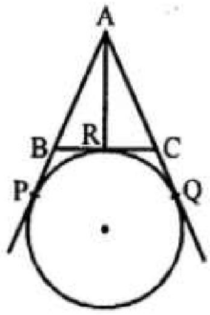
Find the distance between Neeta and top tower CD?

SOLUTION

Section A

1. **(a)** 7.5 cm

Explanation: In the given figure, AP, AQ and BC are tangents to the circle.



AB = 5 cm, AC = 6 cm, BC = 4 cm

Length of AP = ?

BP and BR are the tangents to the circle.

BP = BR

Similarly, CR and CQ are tangents

CR = CQ

Similarly, AP and AQ are tangents

AP = AQ

AP = AB + BP = AB + BR

AQ = AC + CQ = AC + CR

AP + AQ = AB + BR + AC + CR = AB + BR + CR + AC

AP + AP = AB + BC + AC

2AP = 5 + 4 + 6 = 15 cm

AP = $\frac{15}{2}$ = 7.5 cm

2. **(d)** ± 4

Explanation: Distance between (4, p) and (1, 0) = 5

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$

$\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$

$\sqrt{(-3)^2 + (-p)^2} = 5$

Squaring, both sides

$(-3)^2 + (-p)^2 = (5)^2 \Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 25 - 9 = 16$

$\therefore p = \pm\sqrt{16} = \pm 4$

3. **(a)** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Explanation: we know that the midpoint formula = $\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

4. **(a)** $\frac{1}{2}$

Explanation: Possible outcomes of tossing three coins are:
(HHH), (HHT), (HTH), (THH), (TTT), (TTH), (THT),(HTT)
here H and T are denoted for Head and Tail.
Total outcomes = 8
no. of outcomes with at least two heads = 4
$\therefore$ required probability $= \frac{4}{8} = \frac{1}{2}$

5. **(c)** $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

Explanation: If the point R(x, y) divides the join of P(x_1, y_2) and Q(x_2, y_2) internally in the given ratio $m_1 : m_2$,
then the coordinates of the point R are $\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$

6. **(a)** parallel

Explanation: Given: a_1 = 6, a_2 = 2, b_1 = -3, b_2 = -1, c_1 = 10 and c_2 =9
$a_1 = 6, a_2 = 2, b_1 = -3, b_2 = -1, c_1 = 10$ and $c_2 = 9$
Here $\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$
but $\frac{c_1}{c_2} = \frac{10}{9}$
$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Therefore, the lines are parallel.

7. **(c)** $\frac{1}{2}$

Explanation: No. of total letters in the word MOBILE = 6
No, of vowels = {O,I,E}, i, e = 3
Probability of being a vowel = $\frac{3}{6} = \frac{1}{2}$

8. **(c)** 1 : 2

Explanation: $2\pi r^2 = \pi rl \Rightarrow \frac{r}{l} = \frac{1}{2}$

9. **(c)** $\frac{1}{3}$

Explanation: Total number of lottery tickets = 8 + 16 = 24.
Number of prizes = 8.
$\therefore$P (getting a prize) = $\frac{8}{24} = \frac{1}{3}$

10. **(c)** $\alpha + \beta = \frac{b}{a}$

Explanation: If α and β are the roots of $ax^2 + bx + c = 0$,
then $\alpha + \beta = \frac{-b}{a}$

11. **(b)** Real and Distinct roots

Explanation: D =b^2 - 4ac
D = $(-2)^2$ - 4 $\times$ 4 $\times$ (-3)
D = 4 + 48
D = 52
D > 0. Hence Real and Distinct roots.

12. **(d)** $\frac{1}{3}$

Explanation: Given: $\sqrt{3}\tan\theta = 3\sin\theta$
$\Rightarrow \sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\Rightarrow \frac{\sqrt{3}}{3} = \cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

And $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$

$\therefore \sin^2\theta - \cos^2\theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

13. **(c)** x-axis

Explanation: Since the ordinates of given points are 0. Therefore, points lie on x-axis.

14. **(b)** a rational number

Explanation: It can be expressed in $\frac{p}{q}$ form

$2.35 = \frac{235}{100}$

so, 2.35 is a rational number

15. **(c)** median

Explanation: A cumulative frequency distribution is the sum of the class and all classes below it in a frequency distribution. Construction of cumulative frequency table is useful to determine Median.

16. **(c)** decreasing

Explanation:

If the elevation moves towards the tower, it is increasing and if its elevation moves away from the tower, it decreases. Hence if the shadow of a tower is increasing, then the angle of elevation of the sun is not increasing.

17. **(d)** 3, 420

Explanation: We have,

$12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 5 \times 3$

HCF = 3

and L.C.M = $2 \times 2 \times 3 \times 5 \times 7$

= 420

18. **(d)** A is false but R is true.

Explanation: $\frac{3072}{16} = 192 \neq 162$

19. **(d)** 6

Explanation: For non-zero solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{k} = \frac{5}{10} = \frac{0}{0}$

Taking, $\frac{3}{k} = \frac{5}{10} \Rightarrow k = \frac{3\times10}{5} = 6$

20. **(d)** A is false but R is true.

Explanation: Similar triangles are not always congruent.

Section B

21. i. Total number of favourable outcomes = 20

$Probablity\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

Number of favourable outcomes = 4

Hence P (getting a defective bulb) = $\frac{4}{20} = \frac{1}{5}$

ii. Now total number of favourable outcomes = 20 - 1 = 19

Number of favouroable outcomes = 19 - 4 = 15

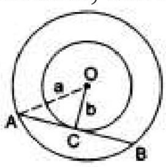
Hence P (getting a non-defective bulb) = $\frac{15}{19}$

22. Let the present age of Aftab and his daughter be x and y years respectively. Then, the pair of linear equations that represent the situation is
$x - 7 = 7(y - 7)$, i.e., $x - 7y + 42 = 0$ ...(1)
and $x + 3 = 3(y + 3)$, i.e., $x - 3y = 6$ ...(2)
from equation (2), we get $x = 3y + 6$
By putting this value of x in equation (1), we get
$(3y + 6) - 7y + 42 = 0$,
i.e., $-4y = -48$, which gives $y = 12$
Again by putting this value of y in equation (2), we get
$x = 3 \times 12 + 6 = 42$
So, the present age of Aftab and his daughter are 42 and 12 years respectively.

OR

Let length of rectangular garden = x metres
and width of rectangular garden = y metres
According to given conditions, perimeter = 36 m
$\frac{1}{2}(2x + 2y) = 36$
$\Rightarrow x + y = 36$ ....(i)
and $x = y+4$
$\Rightarrow x - y = 4$ ....(ii)
Adding eq. (i) and (ii),
$2x = 40 \Rightarrow x = 20$ m
Subtracting eq. (ii) from eq. (i),
$2y = 32 \Rightarrow y = 16$ m
Hence, length = 20 m and width = 16 m

23. Let the coordinates of the required point be (x, y). Then,
$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$
$= \frac{(2)(4)+(3)(-1)}{2+3}$
$= \frac{8-3}{5} = \frac{5}{5} = 1$
$y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$
$= \frac{(2)(-3)+(3)(7)}{2+3}$
$= \frac{-6+21}{5} = \frac{15}{5} = 3$
Hence, the required point is (1, 3).

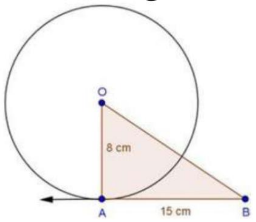
24. 

Let O be the common centre of the two circles
and AB be the chord of the larger circle which touches the smaller circle at C.
Join OA and OC.
Then, OA = a and OC = b.
Now, $OC \perp AB$ and OC bisects AB [$\because$ the chord of the larger circle touching the smaller circle, is bisected at the point of contact].
In right $\triangle ACO$, we have

$OA^2 = OC^2 + AC^2$ [by Pythagoras' theorem]
$\Rightarrow AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}$.
$\therefore AB = 2AC = 2\sqrt{a^2 - b^2}$ [$\because$ C is the midpoint of AB]
i.e, required length of the chord $AB = 2\sqrt{a^2 - b^2}$

OR

Since, tangent at a point on a circle is perpendicular to the radius through the point.



Therefore, OA is perpendicular to AB.
In right triangle OAB, We have
$OB^2 = OA^2 + AB^2$
$\Rightarrow OB^2 = 8^2 + 15^2$
$= 64 + 225$
$= 289$
$\Rightarrow OB^2 = 289$
$\Rightarrow OB^2 = (17)^2$
$\Rightarrow OB = 17$ cm.

25. The number of zeroes is 1 as the graph intersects the x-axis at one point only.

Section C

26. We have $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$
after putting values,we get
$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$
$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$
$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$ Rationalise it, we get
$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$
$= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2}$
$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$
$= \frac{43 - 24\sqrt{3}}{11}$

27. We can prove $7\sqrt{5}$ irrational by contradiction.
Let us suppose that $7\sqrt{5}$ is rational.
It means we have some co-prime integers a and b ($b \neq 0$)
such that
$7\sqrt{5} = \frac{a}{b}$
$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ....(1)
R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.
It is not possible which means our supposition is wrong.
Therefore, $7\sqrt{5}$ cannot be rational.
Hence, it is irrational.

OR

The number of participants in each room must be the HCF of 60, 84 and 108.
In order to find the HCF of 60, 84 and 108, we first find the HCF of 60 and 84 by Euclid's division algorithm:

2	60	84	1
	48	60	
	12	24	2
	(HCF)	24	
		0	
		(Remainder)	

Clearly, HCF of 60 and 84 is 12
Now, we find the HCF of 12 and 108

12	108	9
(HCF)	108	
	0	
	(Remainder)	

Clearly, HCF of 12 and 108 is 12. Hence, the HCF of 60,84 and 108 is 12.
Therefore, in each room maximum 12 participants can be seated.
We have,
Total number of participants = 60 + 84 + 108 = 252
$\therefore$ Number of rooms required $= \frac{252}{12} = 21$.

28. Let the two numbers be x and y (x > y) then, according to the question, the pair of linear equations formed is:
x - y = 26.........(1)
x = 3y.............(2)

Substitute the value of x from equation (2) in equation (1), we get

3y - y = 26

$\Rightarrow \quad 2y = 26$

$\Rightarrow \quad y = \frac{26}{2}$

$\Rightarrow \quad y = 13$

Substituting this value of y in equation (2), we get

x = 3(13) = 39

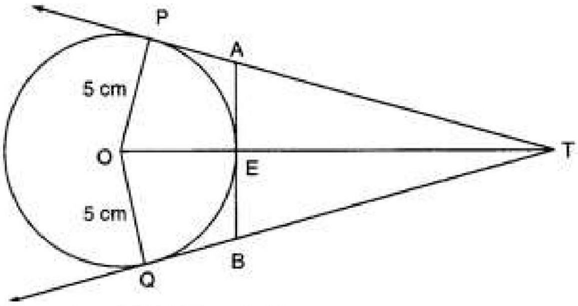
Hence, the required numbers are 39 and 13.

verification: Substituting x = 39 and y = 13, we find that both the equation (1) and (2) are satisfied as shown below:

x - y = 39 - 13 = 26

3y = 3(13) = 39 = x.

This verifies the solution.

29. 

Clearly $\angle OPT = 90°$

Applying Pythagoras in ΔOPT, we have

$OT^2 = OP^2 + PT^2$

$\Rightarrow 13^2 = 5^2 + PT^2$

$\Rightarrow PT^2 = 169 - 25 = 144$

$\Rightarrow$ PT = 12 cm

Since lengths of tangents drawn from a point to a circle are equal. Therefore,

AP = AE = x(say)

$\Rightarrow$ AT = PT - AP = (12 - x)cm

Since AB is the tangent to the circle E. Therefore, $OE \perp AB$

$\Rightarrow \quad \angle OEA = 90°$

$\Rightarrow \quad \angle AET = 90°$ [Applyng Pythagoras Theorem in ΔAET]

$\Rightarrow (12 - x)^2 = x^2 + (13 - 5)^2$

$\Rightarrow 144 - 24x + x^2 = x^2 + 64$

$\Rightarrow$ 24x = 80

$\Rightarrow x = \frac{10}{3}$ cm

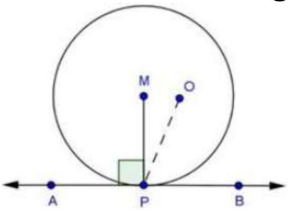
Similarly, BE $= \frac{10}{3}$ cm

$\therefore$ AB = AE + BE = $\left(\frac{10}{3} + \frac{10}{3}\right)$ cm

$= \frac{20}{3}$ cm

OR

Let APB be the tangent and take O as centre of the circle.



Let us suppose that MP⊥AB does not pass through the centre.

Then,

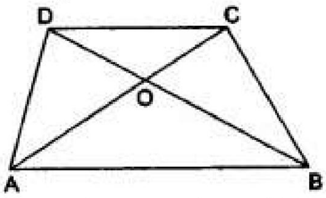
$\angle OPA = 90^\circ$ [∵ Tangent is perpendicular to the radius of circle]

But $\angle MPA = 90^\circ$ [Given]

$\therefore \angle OPA = \angle MPA$

This is only possible when point O and point M coincide with each other.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

30. 

Given A trapezium ABCD in which $AB \| DC$. The diagonals AC and BD intersect at O.

To Prove In $\triangle OAB$ and $\triangle OCD$, we have

$\angle OAB = \angle OCD$ [alternate angles, since $AB \| DC$]

and $\angle OBA = \angle ODC$ [alternate angles, since $AB \| DC$]

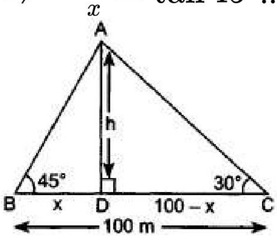
$\therefore \quad \triangle OAB \sim \Delta OCD$ [by AA-similarity].

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

31. In right $\triangle$ADB,

h = x

$\Rightarrow \quad \frac{h}{x} = \tan 45^\circ$...(i)



Now in rt. $\triangle$ADC

$\frac{h}{100-x} = \tan 30^\circ$

Solve for h and x.

$\Rightarrow \frac{h}{100-x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h = 100 - x$

$\Rightarrow \quad \sqrt{3}x = 100 - x$ [Using eq.(i)]

$\Rightarrow (\sqrt{3}+1)x = 100 \Rightarrow x = \frac{100}{\sqrt{3}+1}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{2} = 50(\sqrt{3}-1)$m

∴ h = height of lighthouse = $50(\sqrt{3}-1)$m

Section D

32. Let the usual speed of train be x km/hr

$\frac{300}{x} - \frac{300}{x+5} = 2$

300 (x+5 - x) = 2x(x+5)

150(5) = x^2 + 5x

750 = x^2 + 5x

or, $x^2 + 5x - 750 = 0$

or, $x^2 + 30x - 25x - 750 = 0$

or, (x + 30) (x - 25) = 0

or, x = - 30 or x = 25

Since, speed cannot be negative.

$\therefore \quad x \neq -30, x = 25\text{km/hr}$

$\therefore$ Speed of train = 25 km/hr

OR

Let the original list price be Rs x

$\therefore$ No. of books bought for Rs 300 = $\frac{300}{x}$

Reduced list price of the book = Rs (x - 5)

No. of books bought for Rs 300 = $\frac{300}{x-5}$

According to question,

$\frac{300}{x-5} - \frac{300}{x} = 5$

$\Rightarrow \frac{300x - 300x + 1500}{x^2 - 5x} = 5$

$\Rightarrow x^2 - 5x = 300 \Rightarrow x^2 - 5x - 300 = 0$

$\Rightarrow x^2 - 20x + 15x - 300 = 0$

$\Rightarrow (x - 20)(x + 15) = 0$

$\Rightarrow x = 20 \quad \text{or} \quad x = -15$

$\Rightarrow x = 20$

The negative sign is rejected.

Therefore x = 20

Therefore the original price list is Rs. 20

33. Let the radii of the two circlular plots be r_1 and r_2, respectively.

Then, $r_1 + r_2 = 14$ [∵ Distance between the centres of two circlular plots = 14 cm,given]....(i)

Also, Sum of Areas of the plots = 130π

$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130$...(ii)

Now, from equation (i) and equation (ii),

$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$

$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$

$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$

Solving the quadratic equation we get,

$r_2 = 3$ or $r_2 = 11$,

but from figure it is clear that, $r_1 > r_2$

$\therefore r_1 = 11$ cm and $r_2 = 3$ cm

The value depicted by the farmers are of cooperative nature and mutual understanding.

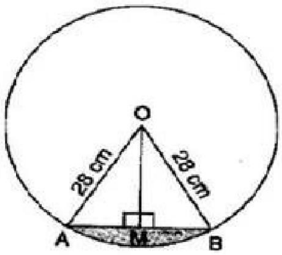
OR

$r = 28$ cm and $\theta = \frac{360}{6} = 60^\circ$

Area of minor sector = $\frac{\theta}{360}\pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$

$= 410.67\ cm^2$

For, Area of $\triangle$AOB,



Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore \triangle OMA \cong OMB$ [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow AM = BM = \frac{1}{2}AB$ and $\angle AOM = \angle BOM = \frac{1}{2}AOB = \frac{1}{2} \times 60^o = 30^o$

In right angled triangle OMA, $\cos 30^{o} = \frac{OM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$

$\Rightarrow$ OM = $14\sqrt{3}$ cm

Also, $\sin 30° = \frac{AM}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AM}{28}$

$\Rightarrow$ AM = 14 cm

$\Rightarrow$ 2 AM = 2 $\times$ 14 = 28 cm

$\Rightarrow$ AB = 28 cm

$\therefore$ Area of $\triangle$AOB = $\frac{1}{2} \times$ AB $\times$ OM = $\frac{1}{2} \times 28 \times 14\sqrt{3} = 196\sqrt{3} = 196 \times 1.7 =$ 333.2 cm^2

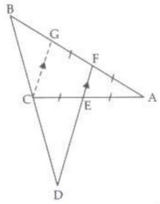
$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle$AOB

= 410.67 - 333.2 = 77.47 cm^2

$\therefore$ Area of one design = 77.47 cm^2

$\therefore$ Area of six designs = 77.47 $\times$ 6 = 464.82 cm^2

Cost of making designs = 464.82 $\times$ 0.35 = Rs. 162.68

34. 

To Prove: $\frac{BD}{CD} = \frac{BF}{CE}$

Construction: Draw CG || EF.

Proof: In $\triangle AGC$ $CG \| EF$

$\because$ E is the mid point of AC

$\therefore$ F will be the mid point of AG.

$\Rightarrow FG = FA$

But, EC = EA = AF [Given]

$\therefore FG = FA = EA = EC$ ....(i)

In $\triangle$BCG and $\triangle$BDF

$EF \| CG$. (By construction)

$\therefore \frac{BC}{CD} = \frac{BG}{GF}$ [By BPT]

$\Rightarrow \frac{BC}{CD} + 1 = \frac{BG}{GF} + 1$

$\Rightarrow \frac{BC+CD}{CD} = \frac{BG+GF}{GF}$

$\Rightarrow \frac{BD}{CD} = \frac{BF}{GF}$

But, FG = CE [From (i)]

$\Rightarrow \frac{BD}{CD} = \frac{BF}{CE}$

Hence, proved.

35.

Class intervals	Frequency (f)	Cumulative frequency (cf/F)
0-100	2	2
100-200	5	7
200-300	x	7 + x
300-400	12	19 + x
400-500	17	36 + x
500-600	20	56 + x
600-700	y	56 + x + y
700-800	9	65 + x + y
800-900	7	72 + x + y
900-1000	4	76 + x + y
		Total = 76 + x + y

We have,

N = Σf_i = 100

$\Rightarrow$ 76 + x + y = 100

$\Rightarrow$ x + y = 24

It is given that the median is 525. Clearly, it lies in the class 500 - 600

$\therefore$ l = 500, h = 100, f = 20, F = 36 + x and N = 100

Now, Median = l + $\frac{\frac{N}{2} - F}{f}$ × h

$\Rightarrow$ 525 = 500 + $\frac{50-(36+x)}{20}$ × 100

$\Rightarrow$ 525 - 500 = (14 - x)5

$\Rightarrow$ 25 = 70 - 5x

$\Rightarrow$ 5x = 45

$\Rightarrow$ x = 9

Putting x = 9 in x + y = 24, we get y = 15

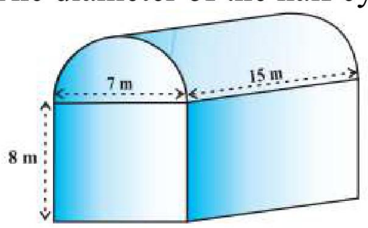
Hence, x = 9 and y = 15

Section E

36. **Read the text carefully and answer the questions:**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.

The diameter of the half cylinder was 7 m and length was 15 m.



(i) Total volume = volume of cuboid + $\frac{1}{2}$ × volume of cylinder.

For cuboidal part we have

length = 15 m, breadth = 7 m and height = 8 m

∴ Volume of cuboidal part = l × b × h $= 15 \times 7 \times 8\text{m}^2 = 840\text{m}^3$

Clearly,

r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2}$m

and, h = Height (length) of half-cylinder = Length of cuboid = 15 m

∴ Volume of half-cylinder = $\frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15\text{m}^3$

$= \frac{1155}{4}\text{m}^3 = 288.75\text{m}^3$

Thus the volume of the air that the shed can hold = (840 + 288.75) m^3 = 1128.75 m^3

(ii) Total space occupied by 20 workers = $20 \times 0.08\text{m}^3$ = = 1.6 m^3

Total space occupied by the machinery = 300 m^3

∴ Volume of the air inside the shed when there are machine and workers inside it

= (1128.75 - 1.6 - 300) m^3

= (1128.75 - 301.6) m^3 = 827.15 m^3

Hence, volume of air when there are machinery and workers is 827.15

(iii) Given for the cuboidal part

length L = 15 m, Width B = 7 m, Height = 8 m

Surface area of the cuboidal part

= 2(LB + BH + HL)

= 2(15 × 7 + 7 × 8 + 8 × 15)

= 2(105 + 56 + 120) = 2 × 281 = 562 m^2

OR

For the cylindrical part r = 3.5 cm and l = 15 m

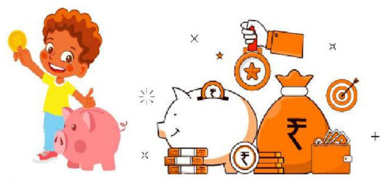
Thus the surface area of the cylindrical part

$= \frac{1}{2}(2\pi rl) = 3.14 \times 3.5 \times 15$

= 164.85 m^2

37. **Read the text carefully and answer the questions:**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1)1] = 190$

$\Rightarrow$ n(n + 1) = 380 $\Rightarrow n^2$ + n - 380 = 0
$\Rightarrow$ (n + 20)(n − 19) = 0 $\Rightarrow$ (n + 20)(n - 19) = 0
$\Rightarrow$ n = −20 or n = 19 $\Rightarrow$ n = -20 or n = 19
But number of coins cannot be negative
$\therefore$ n = 19 (rejecting n = -20)
So, number of days = 19

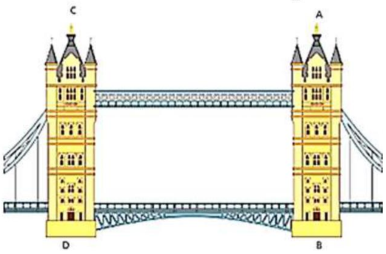
(ii) Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms
$= \frac{19}{2}[2 \times 5 + (19 - 1)5]$
$= \frac{19}{2}[100] = \frac{1900}{2} = 950$
and total money she shaved = ₹950

(iii) Money saved in 10 days
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10-1) \times 5]$
$\Rightarrow S_{10} = 5[10 + 45\}]$
$\Rightarrow S_{10} = 275$
Money saved in 10 days = ₹275

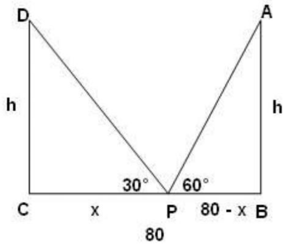
OR

Number of coins in piggy bank on 15th day
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 5]$
$\Rightarrow S_{15} = \frac{15}{2}\ [2 + 14]$
$\Rightarrow S_{15} = 120$
So, there are 120 coins on 15th day.

38. **Read the text carefully and answer the questions:**
Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.
In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60° and 30° respectively.



(i)



Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x).

Also, $\angle APB = 60^{o}$ and $\angle DPC = 30^{o}$

In right angled triangle DCP,

$\tan 30^{o} = \frac{CD}{CP}$

$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{x}{\sqrt{3}}$ ....(i)

In right angled triangle ABP,

$\tan 60^{o} = \frac{AB}{AP}$

$\Rightarrow \frac{h}{80-x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}(80 - x)$

$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$

$\Rightarrow x = 3(80 - x)$

$\Rightarrow x = 240 - 3x$

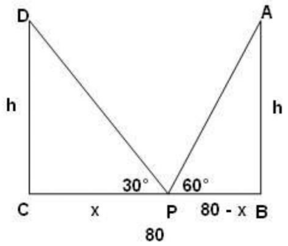
$\Rightarrow x + 3x = 240$

$\Rightarrow 4x = 240$

$\Rightarrow x = 60$

Thus, the position of the point P is 60 m from C.

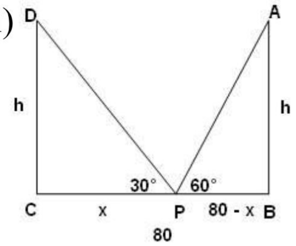
(ii)



Height of the tower, $h = \frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

(iii)



The distance between Neeta and top of tower AB.

In $\triangle ABP$

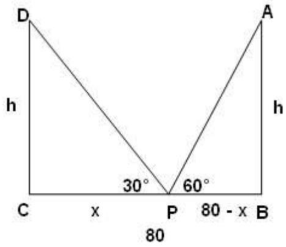
$\sin 60^{o} = \frac{AB}{AP}$

$\Rightarrow AP = \frac{AB}{\sin 60^{0}}$

$\Rightarrow AP = \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AP = 40 m

OR



The distance between Neeta and top of tower CD.

In $\triangle$CDP

$\sin 30^{0} = \frac{CD}{PD}$

$\Rightarrow PD = \frac{CD}{\sin 30^{0}}$

$\Rightarrow PD = \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3}$

$\Rightarrow PD = 40\sqrt{3}$